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Simple Derivation of Minimum Length, Minimum Dipole Moment and Lack of Space–Time Continuity

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The principle of maximum power makes it possible to summarize special relativity, quantum theory and general relativity in one fundamental limit principle each. Special relativity contains an upper limit to speed; following Bohr, quantum theory is based on a lower limit to action; recently, a maximum power given by $c^5/4G$ was shown to be equivalent to the full field equations of general relativity. Taken together, these three fundamental principles imply a limit value for every physical observable, from acceleration to size. The new, precise limit values differ from the usual Planck values by numerical prefactors of order unity. Among others, minimum length and time intervals appear. The limits imply that elementary particles are not point-like and suggest a lower limit on electric dipole values. The minimum intervals also imply that the non-continuity of space–time is an inevitable result of the unification of quantum theory and relativity, independently of the approach used.

KEY WORDS: minimum length; minimum electric dipole moment; maximum acceleration; maximum power; maximum force.

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1. INTRODUCTION

Limit values for physical observables are regularly discussed in the literature. There have been studies of smallest distance and smallest time intervals, as well as largest particle energy and momentum values, largest acceleration values and largest space–time curvature values, among others (Ahluwalia, 1994; Amati *et al.*, 1987; Amelino-Camelia, 1994; Aspinwall, 1994; Doplicher *et al.*, 1994; Garay, 1995; Gross and Mende, 1987; Jaekel and Renaud, 1994; Kempf, 1994a,b; Konishi *et al.*, 1990; Loll, 1995; Maggiore, 1993; Mead, 1964; Ng and Van Dam, 1994; Padmanabhan, 1987; Rovelli and Smolin, 1995; Schön, 1993; Townsend, 1977). Usually, these arguments are based on limitations of measurement apparatuses tailored to measure the specific observable under study. In the following we argue that all these limit statements can be deduced in a simpler

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and more structured manner, by reformulating the main theories of physics themselves as limit statements. The limit value for every physical observable then follows automatically, together with new, corrected numerical prefactors. All such limit statements are shown to follow from the same fundamental principles.

Before we discuss the last principle that allows this procedure, we summarize special relativity and quantum theory as limit principles. We then turn to general relativity, where we show that it can be deduced from a new, equally simple principle.

2. SPECIAL RELATIVITY IN ONE STATEMENT

It is well known that the step from Galilean physics to special relativity can be summarised by a single statement on motion: *There is a maximum speed in nature*. For all physical systems,

$$v \leq c. \tag{1}$$

A few well-known remarks set the framework for the later discussions. The speed v is smaller than or equal to the speed of light for *all* physical systems; in particular, this limit is valid both for composed systems and for elementary particles. The speed limit is valid for all observers. No exception to the statement is known. Only a maximum speed ensures that cause and effect can be distinguished in nature, or that sequences of observations can be defined.

To be clear, in the following we call a *physical system* any region of space–time that contains mass-energy, whose location can be followed over time and which interacts incoherently with its environment. With this definition, shadows and other geometric entities, entangled situations, virtual particles and unbound systems are excluded from the definition of "physical system."

One notes that the other commonly used principle of the theory of special relativity, the equivalence of all inertial observers, is already part of Galilean physics. The essence of special relativity is the constant and invariant maximal speed c.

The opposite statement to the speed limit would be the existence of (long-lived) tachyons. This possibility has been explored and tested in great detail; it leads to numerous conflicts with observations.

The existence of a maximal speed in nature leads to observer-dependent time and space coordinates, to length contraction, time dilation and all the other effects that characterise special relativity. Only the existence of a maximum speed leads to the principle of maximum aging that governs special relativity, and thus at low speeds to the principle of least action.

Special relativity also limits the size of systems, independently of whether they are composed or elementary. Indeed, the speed limit implies that acceleration

a and size l cannot be increased independently without bounds, as the two ends of a system must not interpenetrate (D'Inverno, 1992; Rindler, 2001). The most important case are massive systems, for which

$$l \leq \frac{c^2}{a}. \tag{2}$$

This limit is also valid for the *displacement* d of a system, if the acceleration measured by an external observer is used. This is all textbook knowledge.

3. QUANTUM THEORY IN ONE STATEMENT

Following Bohr (1931, 1961; Jammer, 1974) all of quantum theory can be summarised by a single statement on motion: *There is a minimum action in nature.* For all systems,

$$S \geq \hbar. \tag{3}$$

Also this limit statement is valid both for composite and elementary systems. The action limit starts from the usual definition of the action, $S = \int(T - U)dt$, and states that between two observations performed at times t and $t + \Delta t$, even if the evolution of a system is not known, the action value is at least $\hbar$. (A practical definition of the action in quantum theory is given by Schwinger's quantum action principle.) (Schwinger, 2001) The quantum of action thus expresses the well-known fundamental fuzziness of nature at microscopic scale.

It is easily checked that no observation results in a smaller action value, independently of whether photons, electrons spin 1/2 flips or macroscopic systems are observed. No exception to the statement is known. A minimum action has been observed for fermions, bosons, laser beams, matter systems and for any combination of them. The opposite statement, implying the existence of arbitrary small action values, has been explored in detail; Einstein's long discussion with Bohr, for example, can be seen as a repeated attempt by Einstein to find experiments which allow to measure arbitrary small action values in nature. For every proposal, Bohr found that this aim could not be achieved.

The minimum action value can be used to deduce the uncertainty relation, the tunnelling effect, entanglement, permutation symmetry, the appearance of probabilities, the information theory aspect of quantum theory and the existence of decay and particle reactions. Details of this discussion can be found in various textbooks (Schiller, 1997–2005). Again the minimal action is a constant and invariant limit, valid for all observers.

Obviously, the existence of a minimal or quantum of action was known right from the beginning of quantum theory. The quantum of action is at the basis of all descriptions of quantum theory, including the many-path formulation and the information-theoretic descriptions. The existence of a minimum quantum of action is completely equivalent to all standard developments.

Quantum theory also implies a limit on system size. For a system *at rest*, the action bound $S \leq pd \leq mcd$, together with the quantum of action, implies a limit on the size d of physical systems:

$$d \geq \frac{\hbar}{mc}. \tag{4}$$

In other words, the reduced Compton wavelength is recovered as lower limit to the size of a composite system. However, the limit is *not* valid for the size of *elementary* particles. (We note that we use the term "elementary" in the conventional way; since elementary particles are not-point-like due to the arguments presented here, the attribute can be put into question. For simplicity, we still keep it in this discussion, in the same way that the tern "atom" or "the indivisible" has been kept even after splitting it became possible.)

The minimal action value might surprise at first, especially when one thinks about spin zero particles. However, minimal action is equivalent to a statement on the *total* angular momentum, including the orbital part with respect to the observer. The total observed angular momentum of any physical systems, even of spin 0 particles, is never smaller than $\hbar$.

One notes that by combining the limits (2) and (4) one obtains

$$a \leq \frac{mc^3}{\hbar}. \tag{5}$$

This *maximum acceleration* for systems in which gravity plays no role is discussed in many publications, for example by Caianiello (Caianiello, 1984; Papini, 2002). No experiment has ever reached the limit, despite numerous attempts.

4. GENERAL RELATIVITY IN ONE STATEMENT

Least known of all is the possibility to summarise general relativity in a single principle: *There is a maximum power or force in nature*. For all systems,

$$P \leq \frac{c^5}{4G} = 9.1 \times 10^{51}\,\mathrm{W} \quad \text{and} \quad F \leq \frac{c^4}{4G} = 3.0 \times 10^{43}\,\mathrm{N}. \tag{6}$$

This formulation of general relativity is not yet common. In fact, it has been pointed only 80 years after the general relativity has been around, independently by Gary Gibbons (2002) and the present author (Schiller, 2005, 1997–2004). One notes that the limit statement contains both the speed of light c and the constant of gravitation G; it thus indeed qualifies as a statement on relativistic gravitation. Like the previous limit statements, it is stated to be valid for *all* observers.

The detailed proof of the maximum force principle has been given elsewhere (Gibbons, 2002; Schiller, 2005). It was shown that the maximum power principle is equivalent to the field equations of general relativity. The proof can be summarized in four steps.

(1) Any system that *achieves* the maximum force or power is unattainable and thus must be two-dimensional. Such systems are called horizons.

(2) Since horizons sustain a maximal force or power, they must be curved. They can thus be described by a finite surface gravity a.

(3) The appearance of maximum force or power on horizons is equivalent to the first law of horizon (or black hole) mechanics:

$$E = \frac{c^2}{8\pi G}\, a\, A. \tag{7}$$

(4) The first law of horizon (or black hole) mechanics is equivalent to the field equations of general relativity. To show this, the first law of black hole mechanics is first changed to its differential form

$$\delta E = \frac{c^2}{8\pi G}\, a\, \delta A, \tag{8}$$

and then shown to be equivalent to the field equations by rewriting it in covariant form. To achieve this, one introduces the general surface element $d\Sigma$ and the local boost Killing vector field k that generates the horizon (with suitable norm). Jacobson uses the two quantities to rewrite the left hand side of the first law of horizon mechanics (8) as

$$\delta E = \int T_{ab} k^a \, d\Sigma^b, \tag{9}$$

where T_{ab} is the energy-momentum tensor. This expression obviously gives the energy at the horizon for arbitrary coordinate systems and arbitrary energy flow directions.

Jacobson's main result is that the the right hand side of the first law of horizon mechanics (8) can be rewritten, making use of the (purely geometric) Raychaudhuri equation, as

$$a\, \delta A = c^2 \int R_{ab} k^a \, d\Sigma^b, \tag{10}$$

where R_{ab} is the Ricci tensor describing space–time curvature. This relation thus describes how the local properties of the horizon depend on the local curvature. One notes that the Raychaudhuri equation is a purely geometric equation for manifolds, comparable to the expression that links the curvature radius of a curve to its second and first derivative. In particular, the Raychaudhuri equation does *not* contain any implications for the physics of space–times at all.

Combining these two steps, the first law of horizon mechanics (8) becomes

$$\int T_{ab} k^a d\Sigma^b = \frac{c^4}{8\pi G} \int R_{ab} k^a \, d\Sigma^b. \tag{11}$$

Jacobson then shows that this equation, together with local conservation of energy (i.e., vanishing divergence of the energy-momentum tensor), can only be satisfied if

$$T_{ab} = \frac{c^4}{8\pi G} \left(R_{ab} - \left(\frac{R}{2} + \Lambda \right) g_{ab} \right), \tag{12}$$

where R is the Ricci scalar and Λ is a constant of integration whose value is not specified by the problem. These are the full field equations of general relativity, including the cosmological constant Λ. The field equations thus follow from the first law of horizon mechanics. The field equations are therefore shown to be valid at horizons.

Since it is possible, by choosing a suitable coordinate transformation, to position a horizon at any desired space–time event, the field equations must be valid over the whole of space–time. This conclusion completes the result by Jacobson. Because the field equations follow, via the first law of horizon (or black hole) mechanics, from maximum force, one has thus shown that at every event in nature the same maximum possible force holds; its value is an invariant and a constant of nature.

In other words, the field equations of general relativity are a direct consequence of the limited energy flow at horizons, which in turn is due to the existence of a maximum force (or power). In fact, the argument also works in the opposite direction, since all intermediate steps are equivalences. This includes Jacobson's connection between the horizon equation and the field equations of general relativity. In summary, maximum force or power, the first law of horizon (or black hole) mechanics, and general relativity are thus *equivalent*.

5. MAXIMUM POWER AND EQUIVALENT STATEMENTS

The value of the maximum power is the energy of a Schwarzschild black hole divided by the time that light takes to travel a length equal to twice its radius. The maximum power value $c^5/4G$ is realized when such a black hole is radiated away in the time that light takes to travel along a length corresponding to twice the radius. The value $c^4/4G$ of the force limit is the energy of a Schwarzschild black hole divided by twice its radius.

By dividing the maximum force by the speed of light c, one gets an equivalent limit on the mass change of any physical system:

$$\frac{m}{t} \leq \frac{c^3}{4G}. \tag{13}$$

A further division by c yields an equivalent limit on the ratio between mass and size (diameter) l:

$$\frac{m}{l} \leq \frac{c^2}{4G}. \tag{14}$$

The power limit thus claims that no physical system of a given mass can be concentrated in a region of space–time *smaller* than a (non-rotating) black hole of that mass.

It is easily checked that these upper limits are indeed satisfied by all systems *observed* in nature, whether they are microscopic, macroscopic or astrophysical.

The next aspect to check is whether a system can be *imagined* that exceeds any of these limits. Here the possibilities to be discussed are legion. Most cases have been discussed in detail elsewhere (Schiller, 2005, 1997–2004).

For example, it has been shown that no physical observer can detect force or power values exceeding the limit, despite the apparent divergence of gravitational force at the horizon of black holes.

The force limit is also apparently exceeded in Feynman diagrams, where electrons are supposed to change momentum at a single point of space–time. However, no experimental proof of this possibility exists; indeed, both string theory and quantum gravity resolve the issue by eliminating this point-like change. In nature, any attempt to change high momenta in a short time require an intermediate storage of energy. However, an energy storage cannot take place in a smaller region of space than the diameter of a black hole of that energy during the time of collision. This connection again confirms the force limit. (String theory and quantum gravity also comply with the force and power limits.)

A force limit must be valid for all observers. Even for a moving observer, when the force value is increased by the (cube of the) relativistic dilation factor, or for an accelerating observer, when the observed acceleration is increased by the acceleration of the observer itself, the force limit must still hold. Indeed, when the proper size of observers is taken into account (Schiller, 1997–2004, 2005) no such situation allows to exceed the limit.

The power limit $P \leq c^5/4G = 9.1 \times 10^{51}$ W states that no engine can be more powerful and no radiation source can have a higher luminosity than this value. It is easily checked that in no observation this limit is exceeded. (It was predicted (Schiller, 1997–2004, 2005) the universe itself saturates the limit.) Similarly, it is not possible to imagine a source of particles, light or gravity waves that exceed this limit (Ju *et al.*, 2000; Misner *et al.*, 1973).

Also more concrete attempts to beat the limits inevitably fail. Engines cannot exceed the limit because the exhausts they leave behind at highest powers are so massive that their gravity hinders further acceleration. Attempts to beat the force limit by hanging a mass on a wire and lowering it towards the horizon of a black hole, by building powerful engines that accelerate a car, by using electricity to accelerate a magnetic levitation train, by propelling a rocket through emission of gases, photons or even gravitational waves do not manage to exceed the force (and the power) limit. All these attempts show that the limits hold for gravitational, electromagnetic and nuclear systems. The power, force, mass rate and mass per length limits are not restricted to a specific domain; they are valid for all of nature and for all observers. The limits are constants and invariants of nature.

The last check of the maximum force principle is provided by situations when speeds are much smaller than the speed of light, forces are much smaller than the maximum value and no electromagnetic or similar interaction plays a role. The first condition implies $v \ll c$ and $al \ll c^2$. The second condition requires $\sqrt{4GMa} \ll c^2$. The third condition implies the lack of elementary charge. To be concrete, we take a satellite circling a central mass M at distance R with acceleration a. This system, with length $l = 2R$, has only one characteristic speed. Whenever this speed v is much smaller than c, v^2 must be both equal to $al = 2aR$ and to $\sqrt{4GMa}$. Together, this implies $a = GM/R^2$. In other words, the force limit of nature, applied to systems with low velocities and low curvature, implies the universal law of gravity, as is expected.

In summary, neither of the limits equivalent to general relativity—the limit to force, power, mass rate or mass per length—is or can be exceeded in nature: the limits are found to be valid both in theory and in observation. In addition, each limit is equivalent to the field equations of general relativity (Gibbons, 2002; Schiller, 1997–2004).

6. DEDUCING LIMIT VALUES FOR ALL PHYSICAL OBSERVABLES

The maximum power in nature is equivalent to general relativity and includes universal gravity. As a result, the main physical theories of the twentieth century can be summarized in three simple statements:

$$\begin{aligned} &\text{quantum theory limits action:} && S \geq \hbar \\ &\text{special relativity limits speed:} && v \leq c \\ &\text{general relativity limits power:} && P \leq \frac{c^5}{4G} \end{aligned} \tag{15}$$

Each limit is valid for all physical systems, whether composed or elementary, and is valid for all observers.

When the three fundamental limits are combined using simple algebra, limits for a number of physical observables appear. The following results are valid generally, both for composite and for elementary systems:

$$\text{time interval:} \qquad t \geq \sqrt{\frac{4G\hbar}{c^5}} = 10.4 \times 10^{-44}\ \text{s} \tag{16}$$

$$\text{time distance product:} \qquad td \geq \frac{4G\hbar}{c^4} = 3.4 \times 10^{-78}\,\text{sm} \tag{17}$$

$$\text{acceleration:} \qquad a \leq \sqrt{\frac{c^7}{4G\hbar}} = 2.8 \times 10^{-51}\,\text{m/s}^2 \tag{18}$$

$$\text{angular frequency:} \qquad \omega \leq 2\pi\sqrt{\frac{c^5}{4G\hbar}} = 5.8 \times 10^{43}\,\text{/s} \tag{19}$$

With the additional knowledge that in nature, space and time can mix, one gets

$$\text{distance:} \quad d \geq \sqrt{\frac{4G\hbar}{c^3}} = 3.3 \times 10^{-35}\,\text{m} \tag{20}$$

$$\text{area:} \quad A \geq \frac{4G\hbar}{c^3} = 10.4\times 10^{-70}\,\text{m}^2 \tag{21}$$

$$\text{volume} \quad V \geq \left(\frac{4G\hbar}{c^3}\right)^{3/2} = 3.4 \times 10^{-104}\,\text{m}^3 \tag{22}$$

$$\text{curvature:} \quad K \leq \frac{c^3}{4G\hbar} = 1.0\times 10^{69}\,\text{m}^2 \tag{23}$$

$$\text{mass density:} \quad \varrho \leq \frac{c^5}{16G^2\hbar} = 3.3 \times 10^{95}\,\text{kg/m}^3. \tag{24}$$

Of course, speed, action, force, power, mass rate and mass per length are limited as already stated. Within a numerical factor, for every physical observable the limit corresponds to its Planck value. The limit values can be deduced from the commonly used Planck values simply by substituting G with $4G$. All these limit values are the true *natural units* of nature. In fact, the most aesthetically pleasing solution is to redefine the usual Planck values for every observable to these extremal values by absorbing the numerical factors into the respective definitions. In the following, we call the redefined limits the *(corrected) Planck limits* and assume that the factors have been properly included. In other words, *the natural unit or (corrected) Planck unit is at the same time the limit value of the corresponding physical observable*.

Most of these limit statements are found scattered around the literature, though the numerical prefactors often differ. For example, a smallest measurable distance and time interval of the order of the Planck values are discussed in quantum gravity and string theory (Ng and Van Dam, 1994). A largest curvature has been discussed (Ashtekar, 2005) in quantum gravity. The maximal mass density appears in the discussions on the energy density of the vacuum.

With the present deduction of the limits, two results are achieved. First of all, the various arguments found in the literature are reduced to special cases of three general principles. Second, the confusion about the numerical prefactors is solved. During the history of Planck units, the numerical prefactors have greatly varied. For example, Planck did not include the factors of 2π in the action. And the specialists of relativity did not underline the factor 4 too often. With the present framework, the issue of the correct prefactors in the Planck units should be settled.

Before we discuss the limits in more detail, we complete the list.

7. MASS AND ENERGY LIMITS

A number of observables are missing so far. The remaining observables are related to mass. Mass plays a special role in all these arguments. Indeed, the set of limits (15) does not allow to extract a limit statement on the mass of physical systems through algebraic manipulation. To find a mass limit, the aim has to be restricted.

The Planck limits mentioned so far apply for *all* physical systems, whether they are composed or elementary. Additional limits can only be found if the search is concentrated on *elementary* systems. As shown above, in quantum theory the limit to distance limit is also a limit to size only for *composed* systems. Indeed, a particle is elementary if the particle size l is smaller than the limit size for composed systems:

$$\text{for elementary particles:} \quad l \leq \frac{\hbar}{mc}. \tag{25}$$

By using this new limit, valid only for elementary particles, the well-known mass, energy and momentum limits are found:

$$\begin{aligned} m &\leq \sqrt{\frac{\hbar c}{4G}} = 10.9 \times 10^{-9}\,\text{kg} = 0.59 \times 10^{19}\,\text{GeV}/c^2 \\ E &\leq \sqrt{\frac{\hbar c^5}{4G}} = 9.8 \times 10^{8}\,\text{J} = 0.59 \times 10^{19}\,\text{GeV} \\ p &\leq \sqrt{\frac{\hbar c^3}{4G}} = 3.3\,\text{kg m/s} = 0.59 \times 10^{19}\,\text{GeV}/c \end{aligned} \tag{26}$$

These are elementary particle limits; they are not valid for composed systems. The limits correspond to the corrected Planck mass, energy and momentum, and were already discussed in 1968 by Andrei Sakharov, though again with different numerical prefactors (Sakharov, 1968). They are regularly used and rederived (Wolf, 1994) in elementary particle theory. Obviously, all known measurements comply with the limit values.

8. FAREWELL TO THE CONTINUITY OF SPACE–TIME

The existence of limit values for *every* physical observable has numerous important consequences. Among others, limit values to space–time intervals imply that no physical observable can be described by real numbers; real numbers are approximations (Schiller, 1998).

The most important result might be that three basic limits of nature (15) result in a minimum distance and a minimum time interval. These minimum space and time intervals thus automatically result from the unification of quantum theory and relativity. The limit intervals do not appear if the theories are kept separate.

A limit to space–time measurements implies that it is impossible to say that between two points in space there is always a third one. In fact, it is even impossible to speak about points at all, as the concept of point assumes the ability to reduce length scales to values as small as desired. This is possible on mathematical manifolds, but it is not possible in nature. Therefore, a manifold is not the correct description for space–time.

In other words, *the formulation of physics as a set of limit statements shows that the continuum description of space and time is not correct*. Continuity and manifolds are only approximations valid for large action values, low speeds and low power values.

Fortunately, the difference between space–time and the properties of a manifold is not visible or even measurable in everyday life; such deviations are only apparent in the quantum gravity domain. In nature, this happens only near Planck energies, near the big bang or near horizons. In daily life there is thus no reason to drop the concept of manifold. Dropping it is necessary, however, when quantum gravity effects are explored.

It should be noted that the conclusion about the non-continuity of the vacuum does not depend on the maximum power or force principle. The same result appears for whatever combination of quantum theory and general relativity is used. For example, the same result is found when the consequences of the Compton length and the Schwarzschild radius are combined and studied in detail (Schiller, 1998). These two length scales of physics are sufficient to lead to the non-continuity of space–time. The argument is straightforward: Any physical clock or meter rule is limited by the effects of quantum theory and by effects of general relativity. Obviously, both effects appear together only at energies near the Planck energy. (In daily life, only the quantum effects play a role.) But if extremely high precision or extremely small intervals have to be measured, both these two limitations need to be taken into account at the same time. In those cases one finds both minimum measurement *values* as well as minimum measurement *errors* given by the corrected Planck time or the corrected Planck length (Schiller, 1998).

The result on minimal intervals is stable; has been derived independently in numerous papers, using different methods (Ahluwalia, 1994; Amati *et al.*, 1987;

Amelino-Camelia, 1994; Aspinwall, 1994; Doplicher *et al.*, 1994; Garay, 1995; Gross and Mende, 1987; Jaekel and Renaud, 1994; Kempf, 1994a,b; Konishi *et al.*, 1990; Loll, 1995; Maggiore, 1993; Mead, 1964; Ng and Van Dam, 1994; Padmanabhan, 1987; Rovelli and Smolin, 1995; Schön, 1993; Townsend, 1977). It turns out that it is not important which expression for the clock or ruler limitation one starts with, as long as both quantum theory and general relativity are used in the derivation. Whatever path to quantum gravity is used, one always gets a lower limit to space and time values. In short, nature does show minimum space and time intervals.

Both quantum theory and general relativity assume continuous space–time. Combining them shows that the assumption cannot hold in any combined theory; it can only hold when the two theories are kept separate. Continuity is this not compatible with both of general relativity and quantum theory; it is only compatible with one of the two theories.

The simplest derivation of a minimum length in nature might be the following. General relativity implies a maximum mass change m/t, and quantum theory implies a minimum action S. Now, length l has the following dimensional definition:

$$l^2 = \frac{S}{m/t} \tag{27}$$

As a result, quantum theory with its minimum angular momentum ($S \geq \hbar 2$) and general relativity, with its maximum mass change ($m/t \leq c^3/4G$), imply that length values are limited by

$$l \geq \sqrt{\frac{4G\hbar}{c^3}}. \tag{28}$$

This is the smallest length in nature, the corrected Planck length.

We note that string theory is characterized by a maximum tension, the string tension (Gibbons, 2002). A maximum tension (defined as a negative force) implies, dividing by the speed of light c, a maximum mass change m/t. This explains why string theory—like any other theory that incorporates general relativity and quantum theory—contains a minimum length, and similarly, a minimum time interval.

9. THE ELECTRIC DIPOLE MOMENT OF ELEMENTARY PARTICLES

If the corrected Planck length $\sqrt{4G\hbar/c^3}$ is the smallest size in nature, the smallest electric dipole moment $|D|$ should be given by

$$|D| \geq e\sqrt{\frac{4G\hbar}{c^3}} = 2.1 \times 10^{-51}\,\mathrm{Cm} = 3.3 \times 10^{-35}\,e\mathrm{m}, \tag{29}$$

where e is the charge of the positron. This prediction assumes that charge and mass might not be centred at exactly the same spot in elementary particles. The assumption is based on the idea that a minimum length also implies that no two quantities can be localized at the same spot, given that a minimum length implies that "spots" do not exist in nature.

Naive models of elementary particles, based on the value of the unification energy, expect sizes of about 10^3 times the Planck values, so that actual electric dipole moments might be in the 10^{-32} em range. Given the present measurement limit of 3.4×10^{-29} em (Commins *et al.*, 1994; Akama *et al.*, 2002; Romalis *et al.*, 2001; Lamoreaux, 2001) the electric dipole limit of elementary particles might be testable in not too distant a future. This might be the most accessible quantum gravity effect in nature. One can speculate that, in order to study unification at high energies, a purpose-built facility to measure dipole moments might be a better investment than a new particle collider.

10. OUTLOOK

In summary, we showed that the formulation of twentieth century physics in simple statements implies that in nature every physical observable is limited by a value near the Planck value. In particular, there is a lower limit to size, distance and time measurements. A smallest size for elementary particles suggests a lower limit for the electric dipole moment. This measurement might be possible in the near future. If this is the case, this could be the first measured quantum gravity effect. The existence of limit intervals also implies that the description of space–time with a continuous manifold is not correct, but only an approximation.

On the other hand, the arguments presented so far give no direct hint to the exact microscopic description of space–time. This issue is left for future research.

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